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A NEW APPROACH TO SEPARABILITY AND COMPACTNESS IN SOFT TOPOLOGICAL SPACES

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Abstract. The concept of soft topological space was introduced by some authors. In the present paper, we investigate some basic notions of soft topological spaces by using new soft point concept. Later we give T_i- soft space and the relationships between them are discussed in detail. Finally, we define soft compactness and explore some of its important properties.

Keywords: soft set, soft point, soft topology, soft interior point, soft separation axioms, soft compactness.

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1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. Several theories exist, for example, fuzzy set theory [21], intuitionistic fuzzy set theory [4], rough set theory [16], which can be considered as mathematical tools for dealing with uncertainties. Each of these theories has its inherent difficulties as it was pointed out in [15] that initiated a completely new approach for modeling uncertainties and applied successfully in directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, and so on. [12, 13] defined operations on soft set and gave the first practical application of soft sets in decision making problems. The algebraic structure of set theories dealing with uncertainties is an important problem. Many researchers have contributed towards the algebraic structure of soft set theory [2] defining soft groups and derived their basic properties, [1] introduced initial concepts of soft rings, [7] defined soft semirings and several related notions to establish a connection between soft sets and semirings, [20] defined soft modules and investigated their basic properties. [17] studied soft ideals over a semigroup which characterized generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup. [8, 9] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties.

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[18] firstly introduced the notion of soft topological space which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been by some authors in [3, 6, 10, 11, 14, 19, 22].

In the present study, we first give some basic ideas about soft sets and the results already studied. In addition to these, we give the concept of soft point in [5]. According to this definition, we investigate some important notions of soft topological spaces. Later we give T_i- soft space and the relationships between them are discussed in detail. Finally, we investigate soft compactness and some of its properties.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Molodtsov [15] defined the soft set in the following way. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A \subset E$.

Definition 2.1. [15]. A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \to P(X)$.

Definition 2.2. [13]. For two soft sets (F, A) and (G, B) over X, (F, A) is called a soft subset of (G, B) if

(i) $A \subset B$, and

(ii) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

This relationship is denoted by $(F, A) \subset (G, B)$. Similarly,(F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A) \supset (G, B)$. Two soft sets (F, A) and (G, B) over X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.3. [3]. The soft intersection of two soft sets (F, E) and (G, E) over X is the soft set (H, E), and for all $e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, E) \cap (G, E) = (H, E)$. The soft union of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $H(e) = F(e) \cup G(e)$ for all $e \in E$. This relationship is denoted by $(F, E) \cup (G, E) = (H, E)$.

Definition 2.4. [18]. The complement of a soft set (F, E) is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$ for all $e \in E$.

Definition 2.5. [18]. A soft set (F, E) over X is said to be a null soft set, denoted by Φ, if for all $e \in E$, $F(e) = \emptyset$.

Definition 2.6. [18]. Let Y be a non-empty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$ for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.7. [18]. Let (F, E) be a soft set over X and Y be a non-empty subset of X. Then the soft subset of (F, E) over Y denoted by $\left({}^{Y}F, E\right)$, is defined as follows ${}^{Y}F(e) = Y \cap F(e)$ for all $e \in E$. In other words $\left({}^{Y}F, E\right) = \widetilde{Y} \cap (F, E)$.

Definition 2.8. [18]. Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X if

(1) Φ, $\widetilde{X}$ belong to τ,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open sets in X. A soft set (F, E) over X is said to be a soft closed set in X, if its complement $(F, E)^c$ belongs to τ.

Proposition 2.1. [18]. Let (X, τ, E) be a soft topological space over X. Then the collection $\tau_e = \{F(e) : (F, E) \in \tau \}$ for each $e \in E$, defines a topology on X.

Definition 2.9. [18]. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X . Then the soft closure of (F, E), denoted by $\overline{(F, E)}$,is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Definition 2.10. [18]. Let $x \in X$. Then (x, E) denotes the soft set over X for which $x(e) = \{x\}$ for all $e \in E$.

Definition 2.11. [22]. The soft set (F, A) is called a soft point in X, denoted by e_F, if for the element $e \in A$, $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for all $e' \in A - \{e\}$.

3. Some important concepts of soft topological spaces

Definition 3.1. [5]. Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E) , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$. Let (x_e, E) and $(y_{e'}, E)$ over a common universe X be two soft points. We say that the soft points are different points if $x \neq y$ or $e \neq e'$.

Proposition 3.1. Let (F, E) be a soft set over X. Then (F, E) is the union of its soft points.

Definition 3.2. Let (X, τ, E) be a soft topological space over X . A soft set (F, E) in (X, τ, E) is called a soft neighborhood of the soft point $(x_e, E) \in (F, E)$, if there exists a soft open set (G, E) such that $(x_e, E) \in (G, E) \subset (F, E)$.

The neighborhood system of a soft point (x_e, E), denoted by $U(x_e, E)$, is the family of all its neighborhoods.

Theorem 3.1. The neighborhood system $U(x_e, E)$ at (x_e, E) in a soft topological space (X, τ, E) has the following properties:

1) If $(F, E) \in U(x_e, E)$, then $(x_e, E) \in (F, E)$,

2) If $(F, E) \in U(x_e, E)$ and $(F, E) \subset (G, E)$, then $(G, E) \in U(x_e, E)$,

3) If $(F_1, E), (F_2, E) \in U(x_e, E)$, then $(F_1, E) \cap (F_2, E) \in U(x_e, E)$,

4) If $(F, E) \in U(x_e, E)$, then there exists a $(G, E) \in U(x_e, E)$ such that $(F, E) \in U(y_{e'}, E)$, for each $(y_{e'}, E) \in (G, E)$.

Definition 3.3. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X . The soft point $(x_e, E) \in (F, E)$ is called a soft interior point of a soft set (F, E), if there exists a soft open set $(G, E) \in U(x_e, E)$ such that $(x_e, E) \in (G, E) \subset (F, E)$. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X .Then the soft interior of (F, E), denoted by $(F, E)^{\circ}$, is the union of all soft open subsets of (F, E).

Theorem 3.2. Let (X, τ, E) be a soft topological space and (F, E) be a soft open set over X . Then

$$(F, E) = \bigcup_{e \in E} \{(x_e, E) : (x_e, E) \text{ is any soft interior point of } (F, E) \text{ for each } e \in E\}.$$

Proposition 3.2. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X . Then (F, E) is a soft open set if and only if (F, E) is a soft neighborhood of its soft points.

Definition 3.4. Let (X, τ, E) be a soft topological space, (F, E) be a soft set over X and (x_e, E) be a soft point. Then (x_e, E) is said to be a soft tangency point of (F, E) if $(F, E) \cap (G, E) \neq \Phi$ for arbitrary $(G, E) \in U(x_e, E)$.

Theorem 3.3. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X . Then (F, E) is a soft closed set in X if and only if every soft tangency point of (F, E) belongs to it.

Proof. Let (F,E) be a soft closed set, (x_e,E) be a soft tangency point and $(x_e,E)\notin(F,E)$. Then $(x_e,E)\in(F,E)^c$. Since $(F,E)^c$ is a soft open set in τ, it is a soft neighborhood of (x_e,E). Then $(F,E)^c\cap(F,E)=\Phi$. It follows that $(x_e,E)\in(F,E)$.

Conversely, $(x_e,E)\in(F,E)^c$ be any soft point. Then $(x_e,E)\notin(F,E)$. Since (x_e,E) is not a soft tangency point of (F,E), there exists a soft neighborhood (G,E) of (x_e,E) such that $(F,E)\cap(G,E)=\Phi$. Since $(x_e,E)\in(G,E)\subset(F,E)^c$, we have that $(F,E)^c$ is a soft open set, i.e. (F,E) is a soft closed set.

Proposition 3.3. Let (X,τ,E) be a soft topological space, (F,E) be a soft set over X and $x\in X$. If (x_e,E) is a soft interior point of (F,E), then x is an interior point of $F(e)$ in (X,τ_e).

Proof. For any $e\in E$, $F(e)\subset X$. If (x_e,E) is a soft interior point of (F,E), then there exists $(G,E)\in\tau$ such that $(x_e,E)\in(G,E)\subset(F,E)$. This means that, $x\in G(e)\subset F(e)$ and $G(e)\in\tau_e$. x is an interior point of $F(e)$ in τ_e.

Proposition 3.4. Let (X,τ,E) be a soft topological space, (F,E) be a soft set over X and $x\in X$. If x is a tangency point of $F(e)$ in (X,τ_e), then (x_e,E) is a soft tangency point of (F,E).

Remark 3.1. The converse of Proposition 3.3 and Proposition 3.4 do not hold in general.

Example 3.1. Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and
$\tau=\left\{\Phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\right\}$, where

$$\begin{aligned} F_1(e_1) &= \{x_1,x_2\},\ F_1(e_2)=\{x_1,x_3\},\ F_2(e_1)=\{x_2\},\ F_2(e_2)=\{x_2,x_3\},\\ F_3(e_1) &= \{x_1,x_2\},\ F_3(e_2)=X,\ F_4(e_1)=\{x_2\},\ F_4(e_2)=\{x_3\}. \end{aligned}$$

Then (X,τ,E) is a soft topological space over X. Thus (F,E) is defined as follows:

$$F(e_1)=\{x_1,x_3\},\ F(e_2)=\{x_1,x_3\}. \tag{1}$$

Then there is not a soft interior point of (F,E). But x_1 and x_3 are interior points of $F(e_2)$ in τ_{e_2} . Here $\tau_{e_2}=\{\{x_1,x_3\},\{x_2,x_3\},\{x_3\},X,\emptyset\}$.

Example 3.2. Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and
$\tau=\left\{\Phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),\ (F_6,E),\ (F_7,E)\right\}$, where

$$\begin{aligned} F_1(e_1) &= \{x_1,x_2\},\ F_1(e_2)=\{x_1,x_2\},\ F_2(e_1)=\{x_2\},\ F_2(e_2)=\{x_1,x_3\},\\ F_3(e_1) &= \{x_2,x_3\},\ F_3(e_2)=\{x_1\},\ F_4(e_1)=\{x_2\},\ F_4(e_2)=\{x_1\},\\ F_5(e_1) &= \{x_1,x_2\},\ F_5(e_2)=X,\ F_6(e_1)=X,\ F_6(e_2)=\{x_1,x_2\},\\ F_7(e_1) &= \{x_2,x_3\},F_7(e_2)=\{x_1,x_3\}. \end{aligned}$$

Then (X,τ,E) is a soft topological space over X. Thus (F,E) is defined as follows:

$$F(e_1)=\{x_1,x_3\},\ F(e_2)=\emptyset.$$

Since $\overline{(F,E)}=(F_2,E)^c$, the soft point (x_{2e_2},E) is a soft tangency point of (F,E). But x_2 is not a tangency point of $F(e_2)$ in τ_{e_2}.

4. Soft separation axioms

Definition 4.1. a) Let (X,τ,E) be a soft topological space over X and $(x_e,E)\neq(y_{e'},E)$. If there exist soft open sets (F,E) and (G,E) such that $(x_e,E)\in(F,E)$ and$(y_{e'},E)\notin(F,E)$ or $(y_{e'},E)\in(G,E)$ and $(x_e,E)\notin(G,E)$,then (X,τ,E) is called a soft T_0-space.

b) Let (X, τ, E) be a soft topological space over X and $(x_e, E) \neq (y_{e'}, E)$. If there exist soft open sets (F, E) and (G, E) such that $(x_e, E) \in (F, E)$, $(y_{e'}, E) \notin (F, E)$ and $(y_{e'}, E) \in (G, E)$, $(x_e, E) \notin (G, E)$,then (X, τ, E) is called a soft T_1-space.

c) Let (X, τ, E) be a soft topological space over X and $(x_e, E) \neq (y_{e'}, E)$. If there exist soft open sets (F, E) and (G, E) such that $(x_e, E) \in (F, E)$, $(y_{e'}, E) \in (G, E)$ and $(F, E) \cap (G, E) = \Phi$, then (X, τ, E) is called a soft T_2-space.

Theorem 4.1. Let (X, τ, E) be a soft topological space over X . Then (X, τ, E) is a soft T_1-space if and only if each soft point is a soft closed set.

Proof. Let (X, τ, E) be a soft T_1-space and (x_e, E) be an arbitrary soft point. We show that $(x_e, E)^c$ is a soft open set. Let $(y_{e'}, E) \in (x_e, E)^c$, $(x_e, E) \neq (y_{e'}, E)$. Since (X, τ, E) is a soft T_1- space, there exists soft open set (G, E) such that $(y_{e'}, E) \in (G, E)$, $(x_e, E) \notin (G, E)$. Then $(y_{e'}, E) \in (G, E) \subset (x_e, E)^c$. This implies that $(x_e, E)^c$ is a soft open set, i.e. (x_e, E) is a soft closed set.

Suppose that for each (x_e, E) is a soft closed set in τ. Then $(x_e, E)^c$ is soft open set in τ. Let $(x_e, E) \neq (y_{e'}, E)$. Thus $(y_{e'}, E) \in (x_e, E)^c$ and $(x_e, E) \notin (x_e, E)^c$. Similarly $(y_{e'}, E)^c$ is a soft open set in τ such that $(x_e, E) \in (y_{e'}, E)^c$ and $(y_{e'}, E) \notin (y_{e'}, E)^c$. Therefore (X, τ, E) is a soft T_1-space over X.

Proposition 4.1. Let (X, τ, E) be a soft topological space over X .

a) If (X, τ, E) is a soft T_0- space, then (X, τ_e) is a T_0- space for each $e \in E$.

b) If (X, τ, E) is a soft T_1- space, then (X, τ_e) is a T_1- space for each $e \in E$.

c) If (X, τ, E) is a soft T_2- space, then (X, τ_e) is a T_2- space for each $e \in E$.

Remark 4.1. a) Every soft T_1- space is a soft T_0- space.

b) Every soft T_2-space is a soft T_1-space.

Example 4.1. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \left\{\Phi, \widetilde{X}, (F, E)\right\}$, where

$$F(e_1) = \{x_1\}, F(e_2) = X.$$

Then (X, τ, E) is a soft topological space over X. (X, τ, E) is a soft T_0-space over X which is not a soft T_1-space.

Note that every soft point is a soft closed set in soft T_2-space. However, this fact is not valid if we consider soft point as in [18].

Theorem 4.2. Let X be finite set. The soft topological space (X, τ, E) is a soft T_2-space over X if and only if every soft point in (X, τ, E) is a soft open set.

Proof. Let $X = \{x_1, x_2, ...x_n\}$ be finite set and (X, τ, E) be a soft T_2-space. For soft points $(x_1, E) \neq (x_2, E)$, we choose $(F_{x_1,x_2}, E), (F_{x_2,x_1}, E) \in \tau$ such that $(x_1, E) \in (F_{x_1,x_2}, E)$, $(x_2, E) \in (F_{x_2,x_1}, E)$ and $(F_{x_1,x_2}, E) \cap (F_{x_2,x_1}, E) = \Phi$. Now for the soft points $(x_1, E) \neq (x_3, E)$ and $(x_2, E) \neq (x_3, E)$, we get $(F_{x_1,x_3}, E), (F_{x_3,x_1}, E), (F_{x_2,x_3}, E), (F_{x_3,x_2}, E) \in \tau$ such that

$$(x_1, E) \in (F_{x_1,x_3}, E), (x_3, E) \in (F_{x_3,x_1}, E), \ (F_{x_1,x_3}, E) \cap (F_{x_3,x_1}, E) = \Phi$$

and

$$(x_2, E) \in (F_{x_2,x_3}, E), (x_3, E) \in (F_{x_3,x_2}, E), \ (F_{x_2,x_3}, E) \cap (F_{x_3,x_2}, E) = \Phi.$$

Then the soft open sets

$$\begin{aligned}
(F_{x_1,x_2,x_3}, E) &= (F_{x_1,x_2}, E) \cap (F_{x_1,x_3}, E), \\
(F_{x_2,x_1,x_3}, E) &= (F_{x_2,x_1}, E) \cap (F_{x_2,x_3}, E), \\
(F_{x_3,x_2,x_1}, E) &= (F_{x_3,x_1}, E) \cap (F_{x_3,x_2}, E),
\end{aligned}$$

the condition

$$(x_1, E) \in (F_{x_1,x_2,x_3}, E), (x_2, E) \in (F_{x_2,x_1,x_3}, E), \ \ (x_3, E) \in (F_{x_3,x_2,x_1}, E)$$

and

$$\begin{aligned}(F_{x_1,x_2,x_3}, E) \cap (F_{x_2,x_1,x_3}, E) &= \Phi,\\ (F_{x_1,x_2,x_3}, E) \cap (F_{x_3,x_2,x_1}, E) &= \Phi,\\ (F_{x_2,x_1,x_3}, E) \cap (F_{x_3,x_2,x_1}, E) &= \Phi\end{aligned}$$

are satisfied, i.e., the soft points $(x_1, E), (x_2, E), (x_3, E)$ have disjoint soft neighborhoods. Thus for the soft points $(x_1, E), ..., (x_n, E)$, we can find the soft sets $(F_i, E) \in \tau$ such that $(x_i, E) \in (F_i, E)$, $(x_j, E) \in (F_j, E)$ and $(F_i, E) \cap (F_j, E) = \Phi$, for each $i \neq j$. It is clear that if $i \neq j$, then $(x_i, E) \notin (F_j, E)$. Then $(x_j, E) = (F_j, E)$.

Conversely, the proof is clear.

Theorem 4.3. Let (X, τ, E) be a soft T_1-space, for every soft point (x_e, E), $(x_e, E) \in (G, E)$ and $(G, E) \in \tau$. If there exists a soft open set (F, E) such that $(x_e, E) \in (F, E) \subset \overline{(F, E)} \subset (G, E)$, then (X, τ, E) is a soft T_2-space.

Proof. Suppose that $(x_e, E) \neq (y_{e\prime}, E)$. Since (X, τ, E) is a soft T_1-space, (x_e, E) and $(y_{e\prime}, E)$ are soft closed sets in τ. Thus $(x_e, E) \in (y_{e\prime}, E)^c$ and $(y_{e\prime}, E)$ is a soft open set in τ. Then there exists a soft open set (F, E) in τ such that

$$(x_e, E) \in (F, E) \subset \overline{(F, E)} \subset (y_{e\prime}, E)^c .$$

Hence we have $(y_{e\prime}, E) \in \overline{(F, E)}^c$, $(x_e, E) \in (F, E)$ and $(F, E) \cap \overline{(F, E)}^c = \Phi$, i.e, (X, τ, E) is a soft T_2-space.

Definition 4.2. Let (X, τ, E) be a soft topological space over X, (F, E) be a soft closed set in X and $(x_e, E) \notin (F, E)$. If there exist soft open sets (G_1, E) and (G_2, E) such that $(x_e, E) \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$, then (X, τ, E) is called a soft regular space. (X, τ, E) is said to be a soft T_3-space if it is soft regular and soft T_1- space.

Remark 4.2. A soft T_3-space is a soft T_2- space.

Theorem 4.4. Let (X, τ, E) be a soft topological space over X. (X, τ, E) is a soft T_3-space if and only if for every $(x_e, E) \in (F, E) \in \tau$, there exists $(G, E) \in \tau$ such that $(x_e, E) \in (G, E) \subset \overline{(G, E)} \subset (F, E)$.

Proof. Let (X, τ, E) be a soft T_3-space and $(x_e, E) \in (F, E) \in \tau$. Since (X, τ, E) is a soft T_3-space for the soft point (x_e, E) and soft closed set $(F, E)^c$, there exists $(G_1, E), (G_2, E) \in \tau$ such that $(x_e, E) \in (G_1, E)$, $(F, E)^c \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Thus, we have $(x_e, E) \in (G_1, E) \subset (G_2, E)^c \subset (F, E)$. Since $(G_2, E)^c$ is a soft closed set, $\overline{(G_1, E)} \subset (G_2, E)^c$.

Conversely, let $(x_e, E) \notin (H, E)$ and (H, E) be a soft closed set. Thus, $(x_e, E) \in (H, E)^c$ and from the condition of the theorem , we have $(x_e, E) \in (G, E) \subset \overline{(G, E)} \subset (H, E)^c$. Then $(x_e, E) \in (G, E)$, $(H, E) \subset \left(\overline{(G, E)}\right)^c$ and $(G, E) \cap \left(\overline{(G, E)}\right)^c = \Phi$ are satisfied, i.e., (X, τ, E) is a soft T_3-space.

Theorem 4.5. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft T_3-space, then (X, τ_e) is a T_3-space, for each $e \in E$.

Proof. Let (X, τ, E) be a soft topological space over X. By Proposition 4.1, (X, τ_e) is a T_1-space. Let $B \subset X$ be a closed set in τ_e and $x \notin B$. From the definition of τ_e, there exists a soft closed set (F, E) and $(x_e, E) \notin (F, E)$ such that $F(e) = B$. Since (X, τ, E) is a soft regular space, there exist soft open sets (G_1, E) and (G_2, E) such that $(x_e, E) \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Thus we have $x \in G_1(e)$, $B \subset G_2(e)$ and $G_1(e) \cap G_2(e) = \emptyset$, i.e. (X, τ_e) is a T_3-space for each $e \in E$.

Now we consider soft T_4-space in [18].

Remark 4.3. A soft T_4-space is a soft T_3-space.

Theorem 4.6. Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is a soft T_4-space if and only if, for each soft closed set (F, E) and soft open set (G, E) with $(F, E) \subset (G, E)$, there exists soft open set (D, E) such that

$$(F, E) \subset (D, E) \subset \overline{(D, E)} \subset (G, E) .$$

Proof. Let (X, τ, E) be a soft T_4-space, (F, E) be a soft closed set and $(F, E) \subset (G, E)$, $(G, E) \in \tau$. Then $(G, E)^c$ is a soft closed set and $(F, E) \cap (G, E)^c = \Phi$. Since (X, τ, E) is a soft T_4-space, there exist soft open sets (D_1, E) and (D_2, E) such that $(F, E) \subset (D_1, E)$, $(G, E)^c \subset (D_2, E)$ and $(D_1, E) \cap (D_2, E) = \Phi$. This implies that

$$(F, E) \subset (D_1, E) \subset (D_2, E)^c \subset (G, E) .$$

$(D_2, E)^c$ is a soft closed set and $\overline{(D_1, E)} \subset (D_2, E)^c$ is satisfied. Thus

$$(F, E) \subset (D_1, E) \subset \overline{(D_1, E)} \subset (G, E)$$

is obtained.

Conversely, let (F_1, E), (F_2, E) be two soft closed sets and $(F_1, E) \cap (F_2, E) = \Phi$. Then $(F_1, E) \subset (F_2, E)^c$. From the condition of theorem, there exists a soft open set (D, E) such that

$$(F_1, E) \subset (D, E) \subset \overline{(D, E)} \subset (F_2, E)^c .$$

So, (D, E), $\overline{(D, E)}^c$ are soft open sets and $(F_1, E) \subset (D, E)$, $(F_2, E) \subset \overline{(D, E)}^c$ and $(D, E) \cap \overline{(D, E)}^c = \Phi$ are obtained. Hence (X, τ, E) is a soft T_4-space.

5. Soft compact spaces

Now, we get soft compact spaces and investigate some of its important properties.

Definition 5.1. [22]. A family Ψ of soft sets is a cover of a soft set (F, E) if

$$(F, E) \subset \bigcup \{(F_i, E) : (F_i, E) \in \Psi, i \in I\} .$$

It is a soft open cover if each member of Ψ is a soft open set. A subcover of Ψ is a subfamily of Ψ which is also a cover.

Definition 5.2. [22]. A soft topological space (X, τ, E) is soft compact space, if each soft open cover of X has a finite subcover.

Theorem 5.1. Any soft closed subset of a soft compact space is a soft compact set.

Proof. Let (X, τ, E) be a soft compact space and (K, E) be a soft closed set in (X, τ, E). Suppose that $U = \{(F_i, E)\}_i$ is any soft open covering of (K, E) in (X, τ, E). Then since (K, E) is soft closed set, $(K, E)^c$ is a soft open set; hence $U \cup \{(K, E)^c\}$ is a soft open covering of (X, τ, E). Since (X, τ, E) is a soft compact space, there exists a finite subcovering $\left\{\left(F_{i_j}, E\right)\right\}_{j=\overline{1,n}}$ such that $(X, E) = \bigcup\limits_{j=1}^{n} \left(F_{i_j}, E\right) \cup (K, E)^c$. Thus $(K, E) \subset \bigcup\limits_{j=1}^{n} \left(F_{i_j}, E\right)$ is obtained, i.e. (K, E) is a soft compact set.

In the following theorems, we use the concept of a soft point as given in Definition 3.1.

Theorem 5.2. Let (X, τ, E) be a soft topological space X . If (X, τ, E) is a soft compact T_2-space, then (X, τ, E) is a soft compact T_4-space.

Proof. Let (X, τ, E) be a soft T_2-space and (F_1, E) and (F_2, E) be two soft closed sets and $(F_1, E) \cap (F_2, E) = \Phi$. For the arbitrary soft points $(x_e, E) \neq (y_{e'}, E)$, $(x_e, E) \in (F_1, E)$ and $(y_{e'}, E) \in (F_2, E)$ .Since (X, τ, E) is a soft T_2-space, there exist soft open sets $\left(G_{1_{x_e, y_{e'}}}, E\right)$

and $\left(G2_{y_{e'},x_e},E\right)$ such that $(x_e,E)\in\left(G1_{x_e,y_{e'}},E\right)$, $(y_{e'},E)\in\left(G2_{y_{e'},x_e},E\right)$ and $\left(G1_{x_e,y_{e'}},E\right)\cap\left(G2_{y_{e'},x_e},E\right)=\Phi$.

Let (x_e,E) be a fixed soft point and $(y_{e'},E)\in(F_2,E)$ be an arbitrary soft point. Then $\left\{\left(G1_{x_e,y_{e'}},E\right)\right\}_{(y_{e'},E)\in(F_2,E)}$ is a family of soft open sets and is an open cover of (F_2,E). Since (F_2,E) is a soft compact set, there exists finite subfamily of this cover such that $(F_2,E)\subset\bigcup\limits_{i=1}^{n}\left(G2_{y_{ie_i},x_e},E\right)$. Thus $\bigcap\limits_{i=1}^{n}\left(G1_{x_e,y_{ie_i}},E\right)$ is a soft open set and $(x_e,E)\in\bigcap\limits_{i=1}^{n}\left(G1_{x_e,y_{ie_i}},E\right)$. If we take as $(G_{x_e,F_2},E)=\bigcap\limits_{i=1}^{n}\left(G1_{x_e,y_{ie_i}},E\right)$ and $(D_{F_2,x_e},E)=\bigcup\limits_{i=1}^{n}\left(G2_{y_{ie_i},x_e},E\right)$, then $(G_{x_e,F_2},E)\cap(D_{F_2,x_e},E)=\Phi$ is satisfied. Hence there exist soft open sets $(G_{x_e,F_2},E),(D_{F_2,x_e},E)$ in τ such that $(x_e,E)\in(G_{x_e,F_2},E)$, $(F_2,E)\subset(D_{F_2,x_e},E)$. Then the family of soft open sets $\{(G_{x_e,F_2},E)\}_{(x_e,E)\in(F_1,E)}$ is an soft open cover of (F_1,E). Since (F_1,E) is a soft compact set, there exists finite subfamily of this cover such that $(F_1,E)\subset\bigcup\limits_{i=1}^{n}(G_{x_i,e_i},E)$, $(F_2,E)\subset\bigcap\limits_{i=1}^{n}\left(D_{F_2,x_{ie_i}},E\right)$ and $\bigcup\limits_{i=1}^{n}(G_{x_i,e_i},E)\cap\bigcap\limits_{i=1}^{n}\left(D_{F_2,x_{ie_i}},E\right)=\Phi$. This means that (X,τ,E) is a soft compact T_4-space.

Note that, this theorem is not valid if we consider soft point as in [18].

Example 5.1. Let $X=\{x_1,x_2,x_3\}$ and $E=\{e_1,e_2,e_3\}$. The soft sets $F_i:E\to P(X)$ on X, for $1\leq i\leq 36$, are defined as follows

$$
\begin{aligned}
F_1(e_1) &= \{x_1\},\ F_1(e_2)=\{x_1\},\ F_1(e_3)=\{x_1\},\\
F_2(e_1) &= \{x_2\},\ F_2(e_2)=\{x_2\},\ F_2(e_3)=\{x_2\},\\
F_3(e_1) &= \{x_3\},\ F_3(e_2)=\{x_3\},\ F_3(e_3)=\{x_3\},\\
F_4(e_1) &= \{x_1,x_2\},\ F_4(e_2)=\{x_1,x_2\},\ F_4(e_3)=\{x_1,x_2\},\\
F_5(e_1) &= \{x_1,x_3\},\ F_5(e_2)=\{x_1,x_3\},\ F_5(e_3)=\{x_1,x_3\},\\
F_6(e_1) &= \{x_2,x_3\},\ F_6(e_2)=\{x_2,x_3\},\ F_6(e_3)=\{x_2,x_3\},\\
F_7(e_1) &= \{x_1,x_3\},\ F_7(e_2)=\{x_2\},\ F_7(e_3)=\{x_1\},\\
F_8(e_1) &= \{x_1,x_3\},\ F_8(e_2)=\{x_1,x_2\},\ F_8(e_3)=\{x_1\},\\
F_9(e_1) &= \{x_1\},\ F_9(e_2)=\emptyset,\ F_9(e_3)=\{x_1\},\\
F_{10}(e_1) &= X,\ F_{10}(e_2)=\{x_2\},\ F_{10}(e_3)=\{x_1,x_2\},\\
F_{11}(e_1) &= \emptyset,\ F_{11}(e_2)=\{x_2\},\ F_{11}(e_3)=\emptyset,\\
F_{12}(e_1) &= \{x_1,x_3\},\ F_{12}(e_2)=\{x_2,x_3\},\ F_{12}(e_3)=\{x_1,x_3\},\\
F_{13}(e_1) &= \{x_3\},\ F_{13}(e_2)=\emptyset,\ F_{13}(e_3)=\emptyset,\\
F_{14}(e_1) &= X,\ F_{14}(e_2)=\{x_1,x_2\},\ F_{14}(e_3)=\{x_1,x_2\},\\
F_{15}(e_1) &= \{x_1\},\ F_{15}(e_2)=\{x_2\},\ F_{15}(e_3)=\{x_1\},\\
F_{16}(e_1) &= \{x_1,x_3\},\ F_{16}(e_2)=X,\ F_{16}(e_3)=\{x_1,x_3\},\\
F_{17}(e_1) &= \{x_1,x_3\},\ F_{17}(e_2)=\emptyset,\ F_{17}(e_3)=\{x_1\},\\
F_{18}(e_1) &= X,\ F_{18}(e_2)=\{x_2,x_3\},\ F_{18}(e_3)=X,\\
F_{19}(e_1) &= \{x_3\},\ F_{19}(e_2)=\{x_2\},\ F_{19}(e_3)=\emptyset,\\
F_{20}(e_1) &= \{x_2,x_3\},\ F_{20}(e_2)=\{x_1,x_3\},\ F_{20}(e_3)=X,\\
F_{21}(e_1) &= X,\ F_{21}(e_2)=\{x_1,x_3\},\ F_{21}(e_3)=X,
\end{aligned}
$$

$$\begin{aligned}
F_{22}(e_1) &= \emptyset,\ F_{22}(e_2)=\{x_1\},\ F_{22}(e_3)=\{x_1\},\\
F_{23}(e_1) &= \{x_2,x_3\},\ F_{23}(e_2)=X,\ F_{23}(e_3)=X,\\
F_{24}(e_1) &= \{x_2\},\ F_{24}(e_2)=\emptyset,\ F_{24}(e_3)=\{x_2\},\\
F_{25}(e_1) &= \{x_2\},\ F_{25}(e_2)=\emptyset,\ F_{25}(e_3)=\{x_1,x_2\},\\
F_{26}(e_1) &= \{x_3\},\ F_{26}(e_2)=\{x_1,x_3\},\ F_{26}(e_3)=\{x_1,x_3\},\\
F_{27}(e_1) &= \{x_2,x_3\},\ F_{27}(e_2)=\{x_3\},\ F_{27}(e_3)=\{x_2,x_3\},\\
F_{28}(e_1) &= \{x_3\},\ F_{28}(e_2)=\emptyset,\ F_{28}(e_3)=\{x_1\},\\
F_{29}(e_1) &= \{x_3\},\ F_{29}(e_2)=\{x_1\},\ F_{29}(e_3)=\{x_1\},\\
F_{30}(e_1) &= \emptyset,\ F_{30}(e_2)=\emptyset,\ F_{30}(e_3)=\{x_1\},\\
F_{31}(e_1) &= \{x_2,x_3\},\ F_{31}(e_2)=\emptyset,\ F_{31}(e_3)=\{x_1,x_2\},\\
F_{32}(e_1) &= \{x_3\},\ F_{32}(e_2)=\{x_3\},\ F_{32}(e_3)=\{x_1,x_3\},\\
F_{33}(e_1) &= \{x_2,x_3\},\ F_{33}(e_2)=\{x_1\},\ F_{33}(e_3)=\{x_1,x_2\},\\
F_{34}(e_1) &= \{x_3\},\ F_{34}(e_2)=\{x_1,x_3\},\ F_{34}(e_3)=\{x_1,x_3\},\\
F_{35}(e_1) &= \{x_2,x_3\},\ F_{35}(e_2)=\{x_3\},\ F_{35}(e_3)=X,\\
F_{36}(e_1) &= \{x_3\},\ F_{36}(e_2)=\{x_2\},\ F_{36}(e_3)=\emptyset.
\end{aligned}$$

Then $\tau=\left\{\Phi,\widetilde{X},(F_1,E),...,(F_{36},E)\right\}$ is a soft topology. The soft topological space (X,τ,E) is a soft compact space and soft T_2-space. But (X,τ,E) is not a soft normal space. Indeed, we consider the soft closed sets $(F,E)=(F_7,E)^c$, $F(e_1)=\{x_2\}, F(e_2)=\{x_1,x_3\}$, $F(e_3)=\{x_2,x_3\}$ and $(G,E)=(F_{20},E)^c$, $G(e_1)=\{x_1\}, G(e_2)=\{x_2\}$, $G(e_3)=\emptyset$.

Here the soft closed sets are disjoint. The soft open sets $(F_{20},E),(F_{21},E),(F_{23},E)$ contain (F,E) and the soft open sets (F_4,E), (F_7,E), (F_8,E), (F_{10},E), (F_{12},E), (F_{14},E), (F_{15},E), (F_{16},E), (F_{18},E) contain (G,E).

But intersection of this soft sets is not null soft set. Thus (X,τ,E) is a soft compact and soft T_2-space, but it is not soft normal space.

Theorem 5.3. Let (X,τ,E) be a soft topological space over X . If (X,τ,E) is a soft T_2-space and (F,E) is a soft compact set, then (F,E) is a soft closed set.

Proof. Let (X,τ,E) be a soft T_2-space and (F,E) be a soft compact set. We will show that $(F,E)^c$ is a soft open set. Consider $(y_e,E)\in(F,E)^c$, this means that $(x_{e'},E)\neq(y_e,E)$ for each $(x_{e'},E)\in(F,E)$. Since (X,τ,E) is a soft T_2-space, there exist soft open sets $(G_{x_{e'},y_e},E),(D_{y_e,x_{e'}},E)$ such that $(x_{e'},E)\in(G_{x_{e'},y_e},E)$, $(y_e,E)\in(D_{y_e,x_{e'}},E)$ and $(G_{x_{e'},y_e},E)\cap(D_{y_e,x_{e'}},E)=\Phi$. Then the family of soft open sets $(G_{x_{e'},y_e},E)$ is an open cover of (F,E). Then there exists a finite subfamily of this cover such that $(F,E)\subset\bigcup\limits_{i=1}^{n}\left(G_{x_{ie'_i},y_e},E\right)$. We consider the family of soft open sets $\left\{\left(D_{y_e,x_{ie'_i}},E\right)\right\}_{i=\overline{1,n}}$ such that $\left(G_{x_{ie'_i},y_e},E\right)\cap\left(D_{y_e,x_{ie'_i}},E\right)=\Phi$, where $(y_e,E)\in\left(D_{y_e,x_{ie'_i}},E\right)$ for each $i=\overline{1,n}$.

If we take $(D_{y_e},E)=\bigcap\limits_{i=1}^{n}\left(D_{y_e,x_{ie'_i}},E\right)$, then (D_{y_e},E) is a soft open neighborhood of (y_e,E) and $(D_{y_e},E)\cap\left(\bigcup\limits_{i=1}^{n}\left(G_{x_{ie'_i}},E\right)\right)=\Phi$. This implies that $(y_e,E)\in(D_{y_e},E)\subset(F,E)^c$, i.e, $(F,E)^c$ is a soft open set. Thus (F,E) is a soft closed set.

Theorem 5.4. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft compact T_2-space, there exists a soft open set (F, E) such that

$$(x_e, E) \in (F, E) \subset \overline{(F, E)} \subset (G, E)$$

for each soft point (x_e, E) and $(x_e, E) \in (G, E)$, where $(G, E) \in \tau$.

Proof. Let (X, τ, E) be a soft topological space. Since (X, τ, E) is a soft compact T_2-space, it is a soft T_4-space. Every soft T_4-space is a soft T_3-space. Thus (X, τ, E) is a soft T_3-space. Let (x_e, E) be a soft point, $(G, E) \in \tau$ and $(x_e, E) \in (G, E)$. Then $(G, E)^c$ is a soft closed set and $(x_e, E) \notin (G, E)^c$. Since (X, τ, E) is a soft T_3-space, there exists soft open sets $(F, E), (D, E)$ such that

$$(x_e, E) \in (F, E)\,, \;\; (G, E)^c \subset (D, E) \;\text{ and }\; (F, E) \cap (D, E) = \Phi.$$

This implies that

$$(x_e, E) \in (F, E) \subset (D, E)^c \subset (G, E)\,.$$

Hence $(D, E)^c$ is a soft closed set and $\overline{(F, E)} \subset (D, E)^c$. Then

$$(x_e, E) \in (F, E) \subset \overline{(F, E)} \subset (G, E)$$

is obtained.

Theorem 5.5. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft T_i-space, then (Y, τ_Y, E) is a soft T_i-space for $i = 0, 1, 2$.

Proof. Let $(x_e, E), (y_{e'}, E) \in (Y, \tau_Y, E)$ such that $(x_e, E) \neq (y_{e'}, E)$. Thus there exist soft open sets (F, E) and (G, E) in X which satisfying conditions of soft T_i-spaces such that $(x_e, E) \in (F, E)$, $(y_{e'}, E) \in (G, E)$. Then $(x_e, E) \in (F, E) \cap \widetilde{Y}$ and $(y_{e'}, E) \in (G, E) \cap \widetilde{Y}$. Also the soft open sets $(F, E) \cap \widetilde{Y}$, $(G, E) \cap \widetilde{Y}$ in τ_Y are satisfying conditions of soft T_i-space for $i = 0, 1, 2$.

Theorem 5.6. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft T_3-space, then (Y, τ_Y, E) is a soft T_3-space.

Proof. By Theorem 5.5, (Y, τ_Y, E) is a soft T_1-space. Let $(x_e, E) \in \widetilde{Y}$ and (F, E) be a soft closed set in $\widetilde{Y}$ such that $(x_e, E) \notin (F, E)$. Since (F, E) is a soft closed set in $\widetilde{Y}$, $(F, E) = \widetilde{Y} \cap (F_1, E)$, for some soft closed set (F_1, E) in X. It is clear that $(x_e, E) \notin (F_1, E)$. As (X, τ, E) is a soft T_3-space, there exist soft open sets (G_1, E) and (G_2, E) such that $(x_e, E) \subset (G_1, E)$, $(F_1, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Then $(G_1, E) \cap \widetilde{Y}$, $(G_2, E) \cap \widetilde{Y} \in \tau_Y$ such that

$$(x_e, E) \in (G_1, E) \cap \widetilde{Y}, (F, E) \subset (G_2, E) \cap \widetilde{Y} \text{ and } \left((G_1, E) \cap \widetilde{Y}\right) \cap (G_2, E) \cap \widetilde{Y} = \Phi.$$

Thus (Y, τ_Y, E) is a soft T_3-space.

Theorem 5.7. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft T_4-space and $\widetilde{Y}$ is a soft closed set of X, then (Y, τ_Y, E) is a soft T_4-space.

Proof. Let (X, τ, E) be a soft T_4-space and $\widetilde{Y}$ be a soft closed set of X . Let (F_1, E) and (F_2, E) be two soft closed sets over $\widetilde{Y}$ such that $(F_1, E) \cap (F_2, E) = \Phi$. As $\widetilde{Y}$ is a soft closed set, (F_1, E) and (F_2, E) are soft closed sets in X. Since (X, τ, E) is a soft T_4-space, there exist soft open sets (G_1, E) and (G_2, E) such that $(F_1, E) \subset (G_1, E)$, $(F_2, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$.Then $(F_1, E) = (G_1, E) \cap \widetilde{Y}$, $(F_2, E) = (G_2, E) \cap \widetilde{Y}$ and $\left((G_1, E) \cap \widetilde{Y}\right) \cap \left((G_2, E) \cap \widetilde{Y}\right) = \Phi$. This implies that (Y, τ_Y, E) is a soft T_4-space.

6. Conclusion

We have introduced soft separation axioms in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later their important properties are investigated. In the end, we have studied soft compact space and constructed relationship between soft separability and compactness in soft topological spaces.

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